Committee on Radio Astronomy Frequencies

# Scaling of Interference Limits given in ITU-R RA.769


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## 1. **Summary**:


Radio interference limits given in ITU-R RA 769 scale with the root of the product of measurement bandwidth and integration time


## 2. Interference threshold calculation

### 2.1 Background of the Radiometer equation

Thermal noise has a Gaussian distribution $p(u,u_0)$ of *amplitudes*, given by its rms amplitude $u_0$.

$$p\left(u,u_0\right) := \frac{e^{-\frac{u^2}{2\cdot u_0^2}}}{u_0\cdot\sqrt{2\cdot\pi}}$$

As a result, the distribution of *noise power* w, with average power $w_0$, is that of the *squares of amplitudes* (with an appropriate scale factor) given by a $\gamma$-distribution

$$p_\gamma(\alpha,\beta,x) := \frac{\beta^\alpha\cdot x^{\alpha-1}\cdot e^{-\beta\cdot x}}{\Gamma(\alpha)}$$

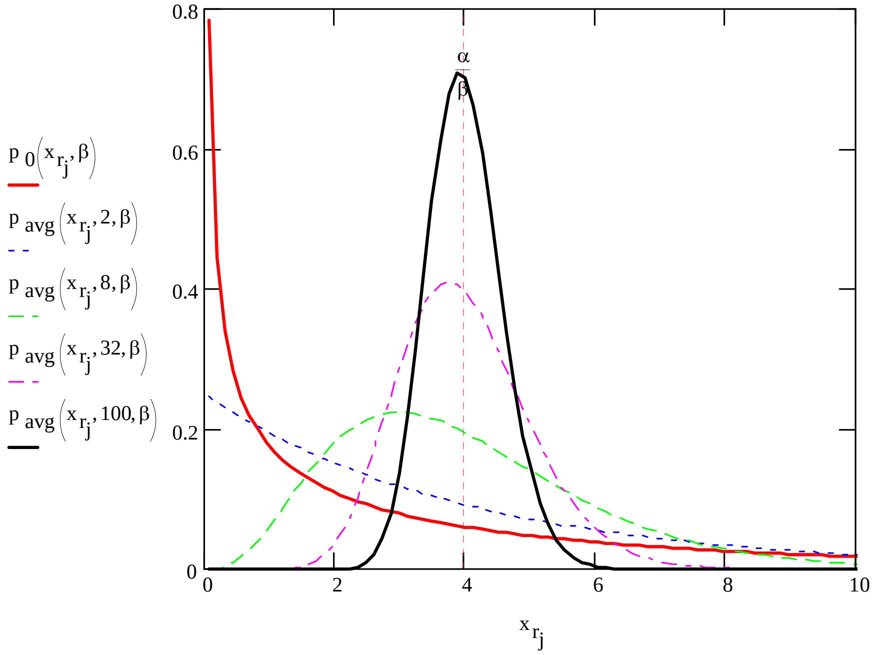
with $x=w$, $\alpha=1/2$ and $\beta=1/2w_0$. The $\gamma$-distribution is not symmetric and defined only for positive values of x!

The mean of a $\gamma$-distribution is $\mu=\frac{\alpha}{\beta}=w_0$ and its variance is $\sigma^2=\frac{\alpha}{\beta^2}=2\cdot w_0^2$ and yields the familiar $2^{1/2}$ factor for the standard deviation of noise power.
(page 475, Gelman et al., Bayesian data analysis, Chapman & Hall, London, 1995).

One can show that the averages x of n noise power measurements also follow a $\gamma$-distribution given for $\beta=1/2w_0$ by

$$p_{avg}(x,n,\beta)=\frac{1}{\Gamma\left(\frac{n}{2}\right)}\cdot x^{\frac{n}{2}-1}\cdot(\beta\cdot n)^{\frac{n}{2}}\cdot e^{-\beta\cdot n\cdot x}$$

The graph illustrates the behaviour of the distribution function for different n:




Increasing the number of averaged measurements leads to a decreasing width, increasing symmetry and closer approach of the peak of the pdf: an illustration of the central limit theorem. But strictly speaking the distribution can never become Gaussian as it is only defined in $x \ \varepsilon \ (0 \ \infty)$. A sufficiently large number of averaged values may narrow the distribution enough that it can be approximated by a Gaussian with negligible errors within a specified interval. There is however no advantage in such an approximation as it does not simply any of the expressions compared to the more familiar Gaussian.

The mean value is again given by $\alpha/\beta$ which evaluates to $w_0$ for $\alpha=n/2$ and $\beta=n/2w_0$.

but the variance yields is now linearly decreasing as $\sigma^2=\frac{2}{n}\cdot w_0$

Hence the standard deviation $\Delta_n$ for the average of n independent noise measurements is given by $\Delta_n=w_0\cdot\sqrt{\frac{2}{n}}$

The Nyquist-Shannon sampling theorem states that a function x(t) that contains no higher frequency components than $\nu$, is fully defined by sampling it at a rate $\tau_S = 1/(2\nu)$. This is a necessary and sufficient condition for the number of independent measurements that can be made for a band-limited signal. The variable output of the described square-law detector has a maximum frequency given by the bandwidth $\Delta\nu$

of the input noise signal. Hence the fastest sampling rate for independent measurements of the output is $\tau_s = \frac{1}{2 \cdot \Delta\nu}$

and the number of independent measurements that are averaged over an integration time $t_{int}$ is then $n = \frac{t_{int}}{\tau_s} = 2 \cdot \Delta\nu \cdot t_{int}$

Using this result in the expression for $\Delta_n$ gives $\Delta_n = \frac{w_0}{\sqrt{\Delta\nu \cdot t_{int}}}$

or more familiar as the sensitivity according to ITU-R RA 769-2, Annex 1, Equation 1

$$\frac{\Delta_n}{w_0} = \frac{\Delta P}{P} = \frac{1}{\sqrt{\Delta f_0 t}} \qquad (1)$$

where:

*P* and $\Delta P$: power spectral density of the noise

$\Delta f_0$: bandwidth

*t*: integration time. *P* and $\Delta P$ in equation (1) can be expressed in temperature units through the Boltzmann's constant, *k*:

$\Delta P = k\ \Delta T$; also $P = k\ T$

(quoted from RA 769) and according to the recommendation, the applicable interference threshold shall be 10% of ΔP). The equation is also known as the 'radiometer equation'.

## 2.2 Scaling with measurement bandwidth and integration time.

Tables 1 and 2 of ITU-R 769-2 provide a convenient access to the interference limits calculated from the radiometer equation.

- Let us use the example of the spectroscopy limit from table 2 for $t_{int} := 2000 \cdot s$ at $\nu := 1612 \cdot MHz$ (column 1) and a bandwidth of $\Delta\nu := 20000 \cdot Hz$ (column 2).
- The next two columns (3&4) give typical receiver ($T_R$= 10 K) and antenna temperatures ($T_A$=12 K), together yielding a system Temperature of 22K.
- Using the radiometer equation for the temperature sensitivity

$$\Delta T := \frac{T_A + T_R}{\sqrt{\Delta\nu \cdot t_{int}}}$$

  yields the entry of 3.479 mK in column 5.

- Multiplication with the Boltzmann constant and division by the measurement bandwidth Δν gives the system sensitivity in terms of power spectral density $\Delta P_s$:

$$10 \cdot \log\left(\Delta T \cdot k \cdot \frac{Hz}{W}\right) = -253.185$$ dB(W/Hz) listed in column 6.

- The interference limit is set to be 10 dB below the sensitivity and the product with the observation bandwidth gives the input power limit in column 7:

$$10\cdot\log\left(\Delta T\cdot k\cdot\frac{Hz}{W}\right)+10\cdot\log\left(\frac{\Delta\nu}{Hz}\right)-10=-220.175 \quad \text{dB(W).}$$

- The power flux density $S_H\Delta\nu$ in column 8 is obtained through division by the isotropic antenna area $\frac{c^2}{4\cdot\pi\cdot\nu^2}$ and is explicitly given by

$$10\cdot\log\left(\Delta T\cdot k\cdot\frac{Hz}{W}\right)+10\cdot\log\left(\frac{\Delta\nu}{Hz}\right)-10\cdot\log\left(\frac{c^2}{4\cdot\pi\cdot\nu^2\cdot m^2}\right)-10=-194.572 \quad \text{dB(W/m}^2\text{)}$$

- The last column (9) gives the emission limit in terms of spectral power flux density spfd and can be had by simply omitting the bandwidth term in the equation above:

$$10\cdot\log\left(\Delta T\cdot k\cdot\frac{Hz}{W}\right)-10\cdot\log\left(\frac{c^2}{4\cdot\pi\cdot\nu^2\cdot m^2}\right)-10=-237.582 \quad \text{dB(Wm}^{-2}\text{Hz}^{-1}\text{)}$$

  Note that the conversion to the radio astronomical unit of *Jansky* is achieved by adding 260 to the value in column 9. Hence the spectroscopy limit for 2000 s integration at 1420 MHz is equivalent to 22.4 dB(Jy) or 174.5 Jy.

The recommendation gives an explicit example (Appendix 1, 1.2) how integration times scale w.r.t. to the reference time of 2000s:

$$10\cdot\log\left(\sqrt{\frac{10\cdot hr}{2000\cdot s}}\right)=6.276 \quad \text{dB}$$

The footnotes to tables 1 and 2 give further examples of the procedure to be used.

It follows from eqn. 1 and the derivation of the table entries, that changes in the *measurement bandwidth* are to be treated in the same manner as changes in integration time. Hence the equation $S_{769}=10^{\frac{-238+260}{10}}\cdot\sqrt{\frac{2000\,\text{sec}}{t_{fin}}\cdot\frac{20kHz}{\delta f}}$ used for the calculation of the thresholds in Leeheim measurements is strictly in accordance with the properties of noise statistics and measurements as outlined in ITU-R RA 769.

### 2.3 Transient sources

The limits in ITU-R RA 769 are given under the assumption of stationary interference. However many sources may occupy only a bandwidth $\Delta\nu_{tr}$ that is smaller than the measurement bandwidth and also transmit their signal for durations $\Delta t_{tr}$ that are much shorter than the measurement time. In these cases, one has to scale the received peak power limit to the measurement bandwidth $\Delta\nu_{rec}$ and integration time $t_{int}$:

$$\Delta P_{Htr} = \Delta P_H \cdot \frac{t_{int}}{\Delta t_{tr}} \cdot \frac{\Delta\nu_{rec}}{\Delta\nu_{tr}}$$

for $\Delta t_{tr} < t_{int}$ and $\Delta\nu_{tr} < \Delta\nu_{rec}$

Times and bandwidths greater than the measurement bandwidth are dealt with in accordance to the procedure in 2.2. It is therefore clear, that even very short duration single transients can be source of interference if they are of sufficient strength. The threshold level for a single interfering radio pulse of duration $\Delta t_{tr}$ and bandwidth $\Delta\nu_{tr}$ increases with integration time and measurement bandwidth:

$$\Delta P_{Htr} = 0.1 \cdot k(T_a + T_r) \cdot \frac{\sqrt{t_{int} \cdot \Delta\nu_{rec}}}{\Delta t_{tr} \cdot \Delta\nu_{tr}}$$

Because of that, the modern high time resolution digital receiver systems used in the search for transient radio astronomical sources will pick up such signals and suffer from interference, even if they happen only infrequently so that they would not have been noticed on longer integrations.

At the Nyquist limit $\frac{\sqrt{t_{int} \cdot \Delta\nu_{rec}}}{\Delta t_{tr} \cdot \Delta\nu_{tr}} = \sqrt{2}$ , and the limit of received peak power is is simply given by $\Delta P_{H\lim} = 0.1 \cdot \sqrt{2} \cdot k(T_a + T_r)$. No averaging takes place and the sensitivity is just at the system noise level, as used in common engineering practise.

Such a peak input power is typically 30 dB higher than the normal input power limit given in column 7 of the tables in ITU-R RA 769, but it can nevertheless be reached by strong pulsed transmissions achievable with modern digital equipment.